Completing squares, $x^4+420x^2+210^2=441x^2+8820x+210^2$, or $(x^2+210)^2=(21x+210)^2$.

Whence, $x^2=21x$, $x=0$, the introduced root, 21, and $\frac{1}{2}[-21\pm\sqrt{(-1239)}]$. Hence, 21 is the only possible real value for x. Therefore, $2940/21=140$, the other factor. Hence 21 and 140 are the factors.

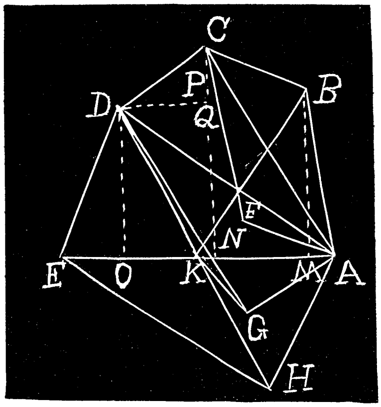
Also solved by H. Prime, V. M. Spunar, S. Lefschetz, J. Scheffer, and S. G. Barton.

GEOMETRY.

377. Proposed by S. A. COREY, Hiteman, Iowa.

Let AB, BC, CD, DE, EA be the sides of a pentagon, plane or gauche. From A draw AF, AG, AH, parallel to, and of the same length and currency as BC, CD, DE, respectively. Bisect AE at K. Draw KB, KF, KG, and KH. Prove that $KB^2+KF^2+KG^2+KH^2=AB^2+BC^2+CD^2+DE^2$.

I. Solution by J. SCHEFFER, A. M., Hagerstown, Md.



K being the mid-point of the diagonal AE in the parallelogram $ADEH$, DKA must be a straight line, viz., the other diagonal. Let the orthogonal coordinates of B, F, G, H, with reference to AE as the axis of X, and A as origin, be, respectively, x_1, y_1; x_2, y_2; x_3, y_3; x_4, y_4; and $AK=EK=a$. Then

$$AB^2+AF^2+AG^2+AH^2=(x_1^2+y_1^2)+(x_2^2+y_2^2)+(x_3^2+y_3^2)+(x_4^2+y_4^2)\ldots\ldots(\mathrm{I}),$$

and $KB^2+KF^2+KG^2+KH^2$

$$=[(x_1-a)^2+y_1^2]+[(x_2-a)^2+y_2^2]+[(x_3-a)^2+y_3^2]+[(x_4-a)^2+y_4^2]$$
$$=(x_1^2+y_1^2)+(x_2^2+y_2^2)+(x_3^2+y_3^2)+(x_4^2+y_4^2)$$
$$-2a[(x_1+x_2+x_3+x_4)-2a]\ldots(\mathrm{II}).$$

Letting fall the perpendiculars BM, CN, DO upon AE, and BP, DQ upon CN, we have $AB\cos BAM+BC\cos CBP+CD\cos CDA+DE\cos DEO=AB\cos BAM+AF\cos FAK+AG\cos KAG+AH\cos KAH=AE=2a$.

$\therefore x_1+x_2+x_3+x_4=2a$; therefore in (II), $x_1+x_2+x_3+x_4-2a=0$, and $AB^2+AF^2+AG^2+AH^2=KB^2+KF^2+KG^2+KH^2=AB^2+BC^2+CD^2+DE^2$. Q. E. D.

II. **Solution by the PROPOSER.**

Let a, b, c, and d be the vector sides AB, BC, CD, and DE, respectively. Then will $EA=-(a+b+c+d)$, $AK=\frac{1}{2}(a+b+c+d)$, $KB=\frac{1}{2}(a-b-c-d)$, $KF=\frac{1}{2}(-a+b-c-d)$, $KG=\frac{1}{2}(-a-b+c-d)$, and $KH=\frac{1}{2}(-a-b-c+d)$.

Squaring these vector expressions for KB, KF, KG, and KH, and adding, their sum is found to be $a^2+b^2+c^2+d^2$. As the square of a vector equals minus the square of its tensor, the truth of the proposition is demonstrated. Observe that a line drawn from D to K is equal to and may be substituted for KH in the equation of the problem.

378. **Proposed by G. I. HOPKINS, A. M., Instructor in Mathematics and Astronomy, Manchester High School Manchester, N. H.**

In the triangle AED, the lines BE and CE are drawn to the points B and C in the base of the triangle. If $AE=100$, $ED=125$, $BC=60$, and $\angle AEC=\angle BED=$a right angle, compute AB, BE, EC, and CD.

Solution by A. H. HOLMES, Brunswick, Maine.

Put $DAE=\theta$ and $ADE=\psi$. Let fall perpendicular EF on base AD. Then we have, $100\sin\theta=125\sin\psi$ or $4\sin\theta=5\sin\psi$...(1).

Since BED and AEC are right angles, $BD=\dfrac{DE}{\cos\psi}$, and $AC=\dfrac{AE}{\cos\theta}$.

$\therefore\ \dfrac{100}{\cos\theta}+\dfrac{125}{\cos\psi}-60=100\cos\theta+125\cos\psi$...(2).

Eliminating $\sin\theta$ and $\cos\psi$ from (1) and (2), and reducing,

$$\cos^6\psi+.96\cos^5\psi-3.1296\cos^4\psi-.0256\cos^3\psi+1.94745\cos^2\psi-.1152\cos\psi=0.$$

Solving by Horner's method, $\cos\psi=.8851+$. $\therefore\cos\theta=.8134+$.

Then since $AC=122.94+$, $AB=62.94+$.

Similarly, $CD=81.22+$. Also, $BE=65.71+$, and $CE=71.51+$.

Also solved by J. Scheffer.

382. **Proposed by PROF. R. C. ARCHIBALD, Brown Uniuersity, Providence, R. I.**

Between the side of a given rhombus and its adjacent side produced, to insert a straight line of a given length and directed to the opposite corner. "Euclidean constructions" are particularly desired.

Remark by V. M. SPUNAR, M. and E. E., Chicago, Illinois.

This is the famous Pappus problem: Rhombo dato et uno latere producto aptare sub angelo exteriori magnitudine datum rectam lineam, quae ad oppositum angulum pertingat.

Pappus, and a certain number of mathematicians, among them Newton, Huygens, and Gergonne, solved the problem algebraically and geometrically. (See E. Pruvost, *Geomètrie Analytique*, t. I, pp. 18-28.)

The problem in the present form, proposed by Prof. R. C. Archibald